Math 1220 Test 4 Name____

Please show **all** appropriate work. Answers that are not justified will not receive full credit. Partial credit will be given when appropriate.

1. Given the function $f(x) = x^{\frac{1}{3}}$ $a = 8$

a) (8 pts) Find a third order Taylor polynomial generated by *f* at *a*.

b) (4 pts) Use the Taylor polynomial in part *a* to approximate $\sqrt[3]{8.4}$. (Round to 2 decimal places)

2. For the power series $\sum \frac{(x-2)^k}{k \cdot 5^k}$

a) (8 pts) Determine the radius of convergence. Show all steps and be precise and mathematically correct in showing how you determine this.

b) (4 pts) Show work for testing the endpoints and determine the interval of convergence.

3. (8 pts) Write the first four terms of the Maclaurin series for $f(x) = \frac{e^{-2x}}{x^2}$. Express these terms without any parenthesis.

4. (10 pts) Use Taylor series to evaluate the limit.

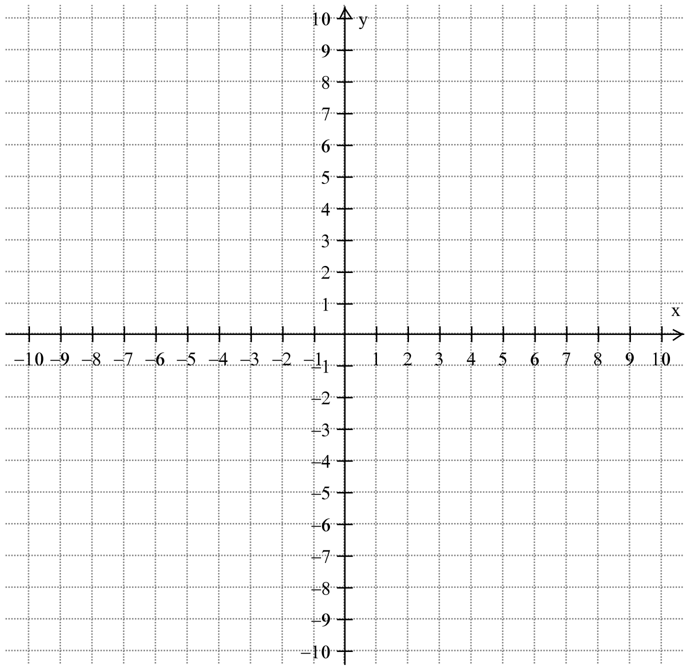
$$\lim_{x\to 0}\frac{2x^2-1+\cos 2x}{x^4}$$

5. (10 pts) Write the first four terms of the binomial series for the given function. Express without parenthesis.

$$f(x)=\ 1+3x\ ^{\frac{1}{3}}$$

6. For the given parametric equations $x = t^2 + 4,\ y = 6 - t,\quad -1 \le t \le 2$

a) (6 pts) Draw the graph and show the orientation (direction).

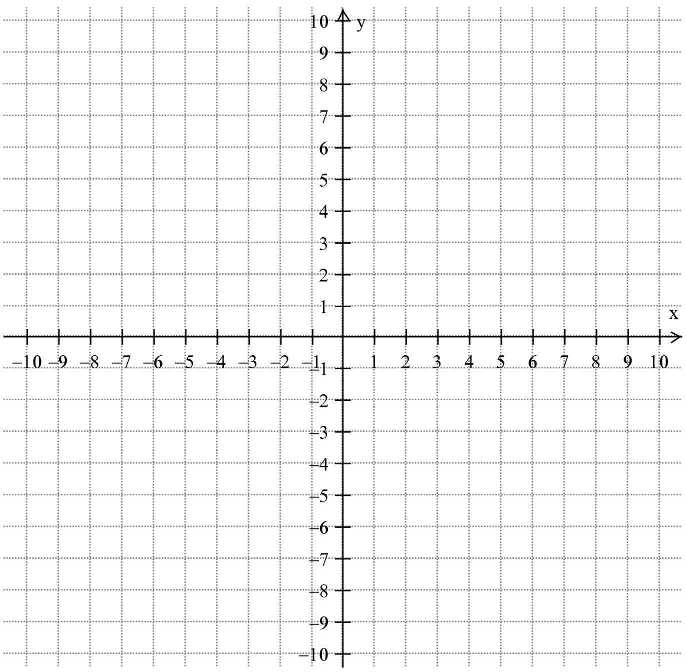


b) (4 pts) Eliminate the parameter to obtain an equation in *x* and *y*.

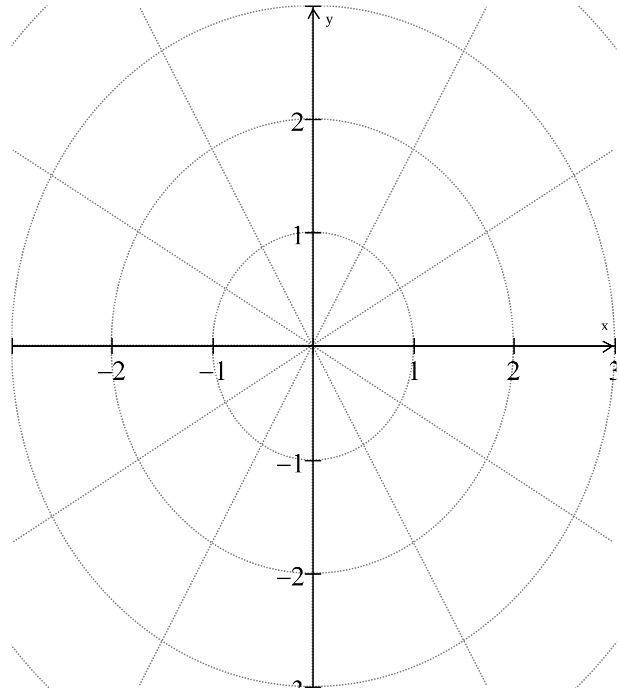
c) (4 pts) Find $\frac{dy}{dx}$ when $t = 1$

7. a) (8 pts) Write the equation $r + 8\sin\theta - 6\cos\theta = 0$ in Cartesian coordinates.

b) (4 pts) Graph the equation you found for Part a above.

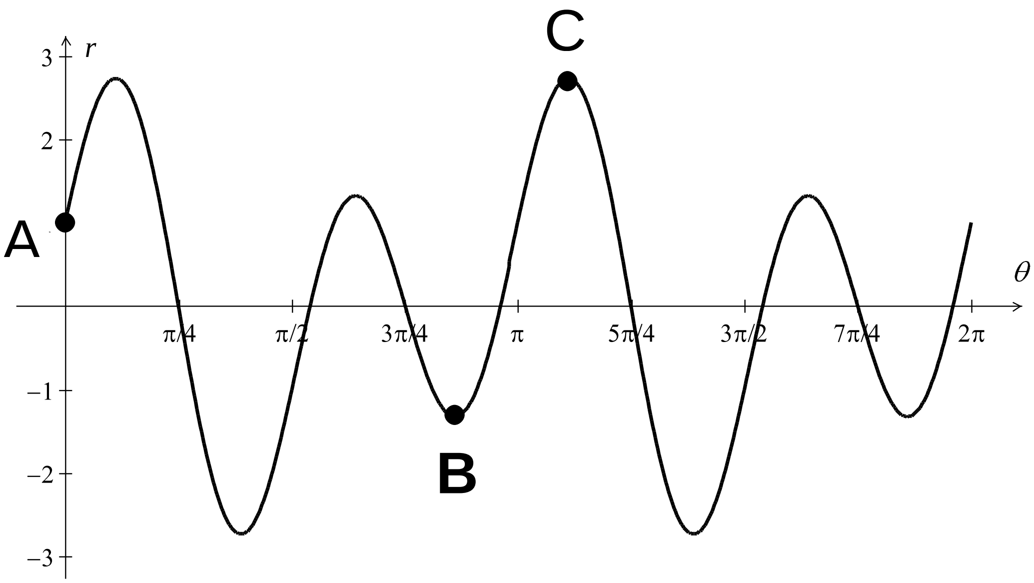


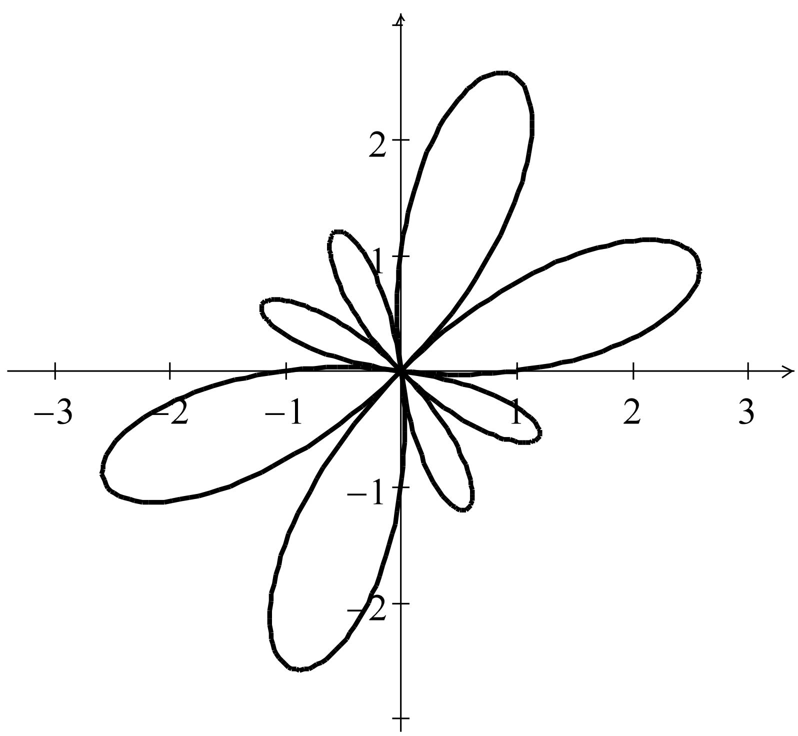
8. a) (8 pts) Graph $r = 2\sin\frac{1}{2}\theta$



b) (8 pts) Find the area inside the figure graphed above. Leave answer in exact form.

9. (6 pts) A Cartesian and a polar graph of $r = \cos 2\theta + 2\sin 4\theta$ are given in the figures below.
Label the points A, B and C on the polar graph that correspond to the points shown on the Cartesian graph.





BONUS: (3 pts) Find a power series solution of the differential equation $y'(t) - 3y(t) = 10$ with initial condition y(0) = 2 . Identify the solution in terms of known functions.