Salt Lake Community College

Calc II Mathematica Mystery Tour

1. Using Mathematica at SLCC

There are at least three ways to use Mathematica at SLCC:

(1) Mathematica is loaded on the machines in the Math Lab.
(2) Mathematica is available through the All Access system. (Go to the main SLCC web page and select "All Access" from the "Current Students" menu.)
(3) You can purchase a personal copy. (Student licenses can be purchased through the Wolfram Research web site for about $100.)

2. Getting Started with Mathematica

The Mathematica system has vast capabilities. It would be impossible to even sketch the full extent of them in an introductory tour. However, it is surprisingly easy to get started with Mathematica.

Please follow these steps to get oriented. Instructions that call for you to enter commands in Mathematica are placed in the red boxes.

1. Open a Notebook:

> Start the Mathematica program. Open a new notebook by clicking the "Notebook" link on the welcome screen, or by selecting `File > New > Notebook` from the application.

2. Evaluating Expressions:

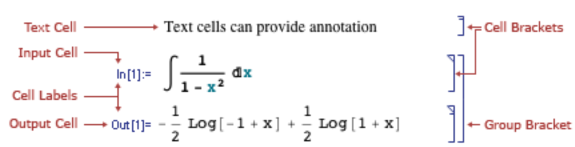
> Position the cursor on the input line and type in
>
> ```
> 7^2
> ```
>
> Then, to evaluate the expression, press `Shift+Enter`.

This should return $7^2 = 49$.

Remark 2.1. It is essential that you hold the shift key while pressing enter. Pressing enter without the shift key simply starts a new line without evaluating.

> Evaluate
>
> ```
> 2 (3 + 1)^2 + 7
> Sin[Pi/3]
> ```

3. Cells: Notice at the right side of the Mathematica notebook window a series of what appear to be brackets.



These are **cell brackets** and **group brackets**. They are used for organizing the work in a notebook. By double clicking on cell brackets, you can show or hide cells.

Each calculation you do is entered in an **input cell** labelled by something of the form

`In[n]:=` $\cdots$

Each calculation Mathematica does is placed in an **output cell** labelled by something of the form

`Out[n]=` $\cdots$

4. Referring to Previous Values:

Evaluate

```
2 % - %%
```

The symbol % in Mathematica refers to the previous computed value (which should be $\sin(\pi/3) = \sqrt{3}/2$) and %% refers to the second to last evaluated value (which is $2(3+1)^2 + 7 = 39$). Therefore, $2\% - \%\% = \sqrt{3} - 39$.

Now evaluate

```
%2
```

This evaluates to the second computed value, which is `Out[2]`. In general, `%n` refers to `Out[n]`.

5. Exact Versus Approximate Values: Mathematica generally only returns exact values unless you explicitly request an approximate decimal value with the numerical approximation function `N[x]`.

Evaluate

```
Log[5]
```

The `Log` function computes natural logarithms. When `Log[5]` is evaluated, Mathematica simply returns `Log[5]`. This is Mathematica's way of returning the exact value $\ln(5)$. If you want the decimal approximation, use the `N` function.

Evaluate

```
N[Log[5]]
```

To get 20 digit accuracy, evaluate

```
N[Log[5], 20]
```

6. Assigning Values:

Type in and evaluate

```
u = 2
v = u^2
w := u^2
```

The variable u now has the value 2 and the variables v and w both currently evaluate to u^2 which is 4. However, the assignment of v uses `=` and is called an **immediate assignment** while the assignment of w uses `:=` and is called a **delayed assignment**. This means that when v is evaluated, the value of u^2 *at the time of assignment* is returned. However, when w is evaluated, the value of u^2 *at the time of evaluation* is returned. In effect, u^2 is re-evaluated each time w is evaluated. To see the distinction,

evaluate

```
{u, v, w}
```

This should return $\{2, 4, 4\}$.

Now evaluate

```
u = 5
```

Then re-evaluate

```
{u, v, w}
```

The result should now be $\{5, 4, 25\}$. Note that the value of w has changed based on the new value of u but v has not.

7. Clearing Values: If you want to remove any assignments from a variable, use the `Clear` function.

Evaluate

```
Clear[u]
```

and then

```
{u, v, w}
```

Note that u now simply evaluates to itself and w evaluates to u^2. In particular, u no longer equals 5.

8. Defining Functions:

Evaluate

```
f[x_] := x^4 + 5
```

This has the effect of defining $f(x) = x^4 + 5$.

Now evaluate

```
f[2]
f[s+2]
f'[x]
```

Remark 2.2. The variable x has an underscore next to it. This makes x a **pattern variable** which can match any expression. It doesn't work to define a function with an ordinary variable like x because in Mathematica x only matches itself.

Evaluate

```
g[x] := x^4 + 5
```

Mathematica takes this to mean that g has value $x^4 + 5$ for argument the symbol x, but not for any other expression.

Evaluate

```
g[2]
g[x]
g[y]
```

9. Capitalization: There is a convention that built in functions in Mathematica, like `Log[x]` and `Sin[x]`, are capitalized. Functions defined by users, like the $f[x]$ we defined above, should be in lower case.

10. Parentheses and Brackets: We have seen that Mathematica treats the different kinds of brackets differently.

( )	groups objects, as in `(5 + 1)^2/(3 + 7)`
[ ]	used for function arguments, as in `Sin[x]`
{ }	used for lists, as in `{2,3,5,7}`

There is one more use for brackets. It is

[[ ]]	used for terms of lists, as in `{2,5,7,9}[[3]]`

which returns 7, the third term of the list.

11. Getting Help: There are several powerful facilities for getting help in Mathematica.

Enter

```
Integrate
```

This is the Mathematica function used to do integration.

> Position your mouse over the command, right click, and select "Get Help."

A new Mathematica window opens with documentation for the `Integrate` command. Note that the provided examples are "live." You can edit them and re-evaluate them, just like any Mathematica code.

> Based on the documentation, enter an appropriate command to compute the definite integral
> $$\int_0^3 \frac{4x-3}{(x^2+9)^2}\,dx.$$

12. Wolfram Alpha: Wolfram Alpha is a technology for evaluating mathematics given in "free-form" natural language. Wolfram Research makes Alpha technology freely available through a web interface. Alpha is also built into Mathematica. To invoke it, click on the "+" beneath the evaluation line where you type commands. From the drop down menu, select "Wolfram—Alpha query." The input changes to a Wolfram Alpha query and any expression evaluated on that line is sent to an Alpha server at Wolfram Research.

> Evaluate
> `sum of first 10 squares`

Another very valuable use of Alpha technology uses "free form" input.

> Click on the "+" beneath the evaluation line. From the drop down menu, select "Free-form input." (The input type changes to free-form.) Evaluate
> `sum of first 10 squares`
> again.

This time, Mathematica indicates the correct way to do this calculation using standard notation, namely `Sum[i^2, {i, 1, 10}]`. In general, you can use this method to quickly locate the correct way to do a calculation in standard notation.

Remark 2.3. You might be tempted to do all your calculations with free-form input. This is not advisable. First, it is slow. Mathematica must communicate with a server on the Internet, which must perform the complex task of making sense of your request. Second, it is not as flexible. It is much easier and more reliable to vary a calculation in standard notation than in free-form.

> Modify the command `Sum[i^2, {i, 1, 10}]` to compute the sum
> $$\sum_{i=1}^{20} \frac{1}{i(i+3)}$$

13. Formatting: Another use of modifying the input type is to introduce headings, comments, and formatting in a Mathematica notebook document.

To place a section heading or other formatting, choose the "other style of text ..." input type. Then in the pop up dialog box, select an appropriate style from among the many choices.

> Select a `Section` type of styled text. Then create a section header titled "Calculus Commands."

To place a comment in a notebook, choose the "Plain text" input type. Then, Mathematica treats what you write as ordinary text and doesn't try to evaluate it.

> Select a plain text input type. Then enter a comment describing how Mathematica will be used to do calculus calculations.

14. Dynamic Interaction: Mathematica can do many kinds of animations and interactive displays. Here, we focus on just the function `Manipulate`.

We'll create an interactive graph of the sine function $f(x) = A\sin(kx)$, where we'll have sliders to set the values of A and k.

Evaluate

```
Plot[2 Sin[4 x],{x,0,2 Pi},PlotRange->{{0,2 Pi},{-3,3}}]
```

This example does the case $A = 2$, $k = 4$. If we want to vary the values of A and k, we can directly edit them and re-evaluate, but there's a much better way.

Evaluate

```
Manipulate[Plot[A Sin[k x],{x,0,2Pi},PlotRange->{{0,2Pi},{-3,3}}],{A,-3,3},{k,0,10}]
```

This creates sliders to control both A and k interactively. For instance, the expression `{A,-3,3}` tells Mathematica to create a slider for A varying its value from -3 to 3. Interact with the graph by using the sliders to change the values of A and h. Then click the small buttons to the right of the sliders to get further control and playback animation buttons. Manipulate these.

We'll make a further modification to our interactive control, but since the expression is getting unwieldy, we can break it into parts as follows. First, turn the plot expression into a function:

Evaluate

```
cmd[A_,k_] := Plot[A Sin[k x],{x,0,2Pi},PlotRange->{{0,2Pi},{-3,3}}]
cmd[2,5]
```

Now apply `Manipulate` to this function.

Evaluate

```
Manipulate[cmd[A,k],{A,-3,3},{k,0,10}]
```

Next, we enhance the command by giving names and starting values to the variables. We'll label A as Amplitude and give it the starting value 1. We'll label k as Angular Frequency and give it the starting value 1 as well.

Evaluate

```
Manipulate[cmd[A,k],{{A,1,"Amplitude"},-3,3},{{k,1,"Angular Frequency"},0,10}]
```

15. Reference Card: Look over the accompanying reference card ReferenceCard.pdf. It summarizes a number of commands that are especially relevant for Calculus II. Cut and paste any of the commands you want to explore into a Mathematica notebook and experiment with them.

3. Further Information

Wolfram's web site provides extensive online documentation, tutorials, and demonstrations. Visit Wolfram Mathematica Documentation Center.

4. Troubleshooting

Like all computer software programs, Mathematica can be flakey. If you get a cryptic error from Mathematica, here are a few things to try.

(1) The most common source of errors is typos. The slightest copying error can create baffling error messages. It's always better to cut and paste than to copy manually.
(2) If Mathematica responds in a bizarre way to a command that seems correct, it could mean that a symbol that you intend as a variable has already been assigned a value that doesn't make sense in the current context.

Evaluate

```
x=3
Solve[x^2+x==2,x]
```

Mathematica responds with

```
Solve::ivar: 3 is not a valid variable. >>.
```

The `Solve` command would be correct if x were not already assigned a value. Because $x = 3$, Mathematica sees this command as `Solve[3^2+3==2,3]`. The bizarre error message is because Mathematica expects a variable in the second position of a `Solve` command and 3 is not a variable.

(3) If you're unable to resolve an error, try restarting the kernel. This will clear everything from memory, including all variable values, function definitions, and loaded packages, and return Mathematica to its initial state. This can be achieved by selecting `Evaluation > Quit Kernel > Local` from the menu. Another way to achieve the same effect is to evaluate the command `Quit[]`.

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