Trig Identity Handout for Math 1060

(The more stars, the more important the identity.)

***Basic identities:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} \qquad \sec\alpha = \frac{1}{\cos\alpha} \qquad \cot\alpha = \frac{1}{\tan\alpha}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha}$$

***Pythagorean identities:

$$\sin^2\alpha + \cos^2\alpha = 1$$

Be able to recognize and quickly derive:

$$1 + \cot^2\alpha = \csc^2\alpha$$
$$\tan^2\alpha + 1 = \sec^2\alpha$$

** Even-Odd identities:

Cosine and secant are EVEN functions:

$$\cos(-x) = \cos x$$
$$\sec(-x) = \sec x$$

All the rest are ODD functions:

$$\sin(-x) = -\sin x \qquad \csc(-x) = -\csc x$$
$$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$$

Sum and difference identities:

*** $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

Use even-odd to derive the difference identity:

$$\cos(\alpha+(-\beta)) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta)$$

* $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

*** $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Use even-odd to derive the difference identity:

$$\sin(\alpha+(-\beta)) = \sin\alpha\cos(-\beta) + \cos\alpha\sin(-\beta)$$

* $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

Double Angle Identities:

$$\sin(2x) = \sin(x+x)$$
$$= \sin x \cos x + \cos x \sin x \quad \text{(apply sum identity)}$$

* $\boxed{\sin(2x) = 2\sin x \cos x}$

$$\cos(2x) = \cos(x+x)$$
$$= \cos x \cos x - \sin x \sin x \quad \text{(apply sum identity)}$$
$$= \cos^2 x - \sin^2 x$$

* $\boxed{\cos(2x) = \cos^2 x - \sin^2 x}$

$$\cos^2 x - \sin^2 x$$
$$= 1 - \sin^2 x - \sin^2 x \quad \text{(apply Pythagorean identity)}$$
$$= 1 - 2\sin^2 x$$

* $\boxed{\cos(2x) = 1 - 2\sin^2 x}$

$$\cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1 - \cos^2 x) \quad \text{(apply Pythagorean identity)}$$
$$= 2\cos^2 x - 1$$

* $\boxed{\cos(2x) = 2\cos^2 x - 1}$

Half angle identities are really just a different way of looking at double angle identities. Let $u = 2x,$ then $\frac{u}{2} = x$. "Plug in" to the double angle identities then solve for either $\sin\left(\frac{u}{2}\right)$ or $\cos\left(\frac{u}{2}\right)$.

$$\cos u = 1 - 2\sin^2\left(\tfrac{u}{2}\right) \qquad \cos u = 2\cos^2\left(\tfrac{u}{2}\right) - 1$$
$$2\sin^2\left(\tfrac{u}{2}\right) = 1 - \cos u \qquad 2\cos^2\left(\tfrac{u}{2}\right) = 1 + \cos u$$

* $\boxed{\sin^2\left(\tfrac{u}{2}\right) = \dfrac{1-\cos u}{2} \qquad \cos^2\left(\tfrac{u}{2}\right) = \dfrac{1+\cos u}{2}}$ ("Power Reducing"-used in Calculus)

* $\boxed{\sin\left(\tfrac{u}{2}\right) = \pm\sqrt{\dfrac{1-\cos u}{2}} \qquad \cos\left(\tfrac{u}{2}\right) = \pm\sqrt{\dfrac{1+\cos u}{2}}}$ Half Angle Identities

* $\boxed{\sin^2(x) = \dfrac{1-\cos 2x}{2} \qquad \cos^2(x) = \dfrac{1+\cos 2x}{2}}$ ("Power Reducing"-used in Calculus)