

Matrix Operations & the Leontief Model

Names _____

Use an Excel spreadsheet to do matrix operations

Example 1: The following tables show the amount of money, in \$1000, earned or lost by men and women in 2001 and 2002.

Table A: Money earned or lost in stocks during 2001

	Men	Women
Tech stocks	\$2000	-\$1000
Pharmaceuticals	\$3000	-\$2000
Energy stocks	\$1000	\$3000

Table B: Money earned or lost in stocks during 2002

	Men	Women
Tech stocks	\$1000	\$2000
Pharmaceuticals	-\$2000	\$4000
Energy stocks	-\$1000	\$3000

Example: Make a table showing the total money earned or lost during the years 2001 – 2002.

Addition:

Let $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -2 & 4 \\ -1 & 3 \end{bmatrix}$, then use Excel to find $A + B$. Enter the values in two 3×2

arrays such as:

	A	B	C	D	E	F	G
1	\$1,000	earned or	lost in	stocks			
2	2001	Men	Women	2002	Men	Women	
3	Tech	2	-1	Tech	1	2	
4	Drugs	3	-2	Drugs	-2	4	
5	Energy	1	3	Energy	-1	3	
6							

It's always nice to label the rows, columns, and matrices as in the example above.

Because the answer will also be a 3×2 array, select a 3×2 array where you want the answer to be by highlighting it, such as:

	A	B	C	D	E	F	G
1	\$1,000	earned or	lost in	stocks			
2	2001	Men	Women	2002	Men	Women	
3	Tech	2	-1	Tech	1	2	
4	Drugs	3	-2	Drugs	-2	4	
5	Energy	1	3	Energy	-1	3	
6	'01 + '02	Men	Women				
7	Tech						
8	Drugs						
9	Energy						

In the formula bar above the spreadsheet which has the function symbol f_x outside the box at the left in newer versions of Excel, or an equal sign = to the left of the box in the older versions, enter the formula: =B3:C5+E3:F5

✓ **Do not hit Enter key, but hit all three keys: Ctrl, Shift, and Enter at the same time.**

3 1

The answer 1 2 should appear in the space selected.

0 6

Multiplication:

Example 2: The first thing to remember when multiplying matrices is that the number of rows in the first matrix must be the same as the number of column in the second matrix; therefore, we cannot multiple matrices $A \times B$ from the previous matrix addition because they are both 3×2 arrays. If we transpose one of the matrices to a 2×3 matrix, then we can multiply them.

If you are using a recent version of Excel to transpose the rows and columns in the second matrix, select and copy the second matrix (for example, E2:F4), then select a cell on the spreadsheet where the row-1, column-1 cell of the transposed matrix will be (for example B6). Then click the down-triangle ▼ to the left of the paste icon in the tool bar, and select Transpose in the drop menu.

If you are using an older version of Excel, select a 2×3 array in which to display the 3×2 matrix you wish to transpose (for example B6:D7), then in the formula bar enter =TRANSPOSE(E2:F4). **Finally hit all three keys: Ctrl, Shift, and Enter at the same time.**

	A	B	C	D	E	F	G
1							
2		2	-1		1	2	
3		3	-2		-2	4	
4		1	3		-1	3	
5							
6		1	-2	-1			
7		2	4	3			

When we multiply the matrices, we must also remember that the answer will have the same number of columns as the first matrix and the same number of rows as the second matrix. So, if we are going to multiply $A \times B^T$, (i.e. matrix A times matrix B transposed), select a 3×3 array to displace the result of the multiplication (for example D9:E10), then in the formula bar, enter either =MMULT(B2:C4,B6:D7) or =MMULT(B2:C4,TRANSPOSE(E2:F4)). The two equations are equivalent since $B6:D7 = \text{TRANSPOSE}(E2:F4)$. Finally hit all three keys: Ctrl, Shift, and Enter at the same time.

	A	B	C	D	E	F	G
1							
2		2	-1		1	2	
3		3	-2		-2	4	
4		1	3		-1	3	
5							
6		1	-2	-1			
7		2	4	3			
8							
9				0	-8	-5	
10				-1	14	-9	
11				7	10	8	
12							

Assignment:

1. a. The tables below give the number of some species of threatened and endangered wildlife in the United States and in foreign countries in 1996.

United States					
	Mammals	Birds	Retiles	Amphibians	Fishes
Endangered	92	74	14	7	65
Threatened	11	16	19	5	40

Foreign					
	Mammals	Birds	Retiles	Amphibians	Fishes
Endangered	252	178	65	8	11
Threatened	19	6	14	1	0

Write the two matrices on a spreadsheet, use the spreadsheet's matrix operations to write a matrix containing the total number of these species, assuming that the U.S. and foreign species are different.

- b. Use the U.S. and foreign matrices with the spreadsheet's matrix operations to write a matrix showing how many more endangered and threatened species there are in foreign countries than there are in the United States. Write a simple equation showing your matrix operation, and explain what any negative numbers mean.

2. A political candidate plans to use three methods of advertising: newspapers, radio, and cable TV. The cost per ad (in thousands of dollars) for each type of media is given in the left table below. The right table shows the number of ads per month in these three media that are targeted to single people, to married males ages 35–55, and the married females over 65 years of age.

	Cable TV	Radio	News papers
Cost (\$1000)	12	15	5

Target Group	Cable TV	Radio	Newspapers
Singles	30	45	35
Males 35–55	25	32	40
Females 65 +	22	12	30

Write the two matrices on a spreadsheet, and use the spreadsheet’s matrix operations to write a matrix showing the cost of ads for each target group. Explain why the cost matrix should not simply be multiplied by the target group matrix as they are written, and what operation needs to be done so they can be multiplied. Write a simple equation showing your matrix operation.

3. A furniture manufacturer produces three styles of chairs with the number of units of each type of raw material needed for each style given in the table below on the left. The cost in dollars per unit for each of the raw materials is given in the table on the right.

	Style A	Style B	Style C
Wood	5	10	5
Nylon	20	9	10
Velvet	0	0	10
Springs	10	0	10

	Cost (\$/unit)
Wood	15
Nylon	12
Velvet	14
Springs	30

Write the two matrices on a spreadsheet, and use the spreadsheet’s matrix operations to write a matrix showing the price of manufacturing each style of chair. Explain why the style matrix should not simply be multiplied by the cost matrix as written above, and what operation needs to be done so they can be multiplied. Write a simple equation showing your matrix operation.

Use an Excel spreadsheet to work the Leontief Input-Output Model

Economist Wassily W. Leontief (1906 - 1999) was awarded the 1973 Nobel Prize in Economics for the development of the input-output method and for its application to important economic problems. He was born in Saint Petersburg, Russia, and educated at the universities of Leningrad (now St. Petersburg) and Berlin. He enrolled at Leningrad University at the age of 15, and graduated five years later with the title of “Learned Economist.” Two year after that he earned his Ph.D. and immigrated to the United States to join the National Bureau of Economic Research in New York City. In 1931 he began his 35-year tenure as a member of the faculty of Harvard University. Leontief’s Nobel Prize in economics (1973) cited him for his creation of the input-output technique. He defined input as the goods and services that each industry buys from all the other industries, and output as the product an industry sells to the others. By this method, the

variations in the flow of goods from one industry to another can be graphically demonstrated using matrices. Economists use his method to analyze, plan, and predict economic changes. Input-output analysis began in the 1920's when the federal government organized data to get a clearer picture of its economy. As a young professor at Harvard, Leontief developed the primary methods of input-output analysis that later were instrumental in the important Controlled Materials Plan during World War II. After World War II, the Air Force and the Department of Labor funded a project called Project Scoop, which provided a detailed analysis of the economy. Since then, input-output analysis has been regarded as one of the cornerstones of economic theory.

In 1965 Leontief published an account of the U.S. economy based on the statistics from 1958. His analysis examined the interdependence among 81 industrial sectors. A modest version is presented here in which the industries are lumped together into six sectors. Table 1 lists the sectors and some representative industries from each. Notice that it includes most of the articles involved in our everyday lives.

Sector	Abbreviation	Representative industries
Final nonmetal	FN	Apparel, food, drugs, furniture
Final metal	FM	Motor vehicles, aircraft, communications
Basic metal	BM	Iron, steel, heating, plumbing
Basic nonmetal	BN	Livestock, agriculture products, fabrics, lumber
Energy	E	Electricity, gas, water, coal, petroleum
Services	S	Transportation, insurance, real estate, finance

The following table is a 6×6 technology matrix that gives Leontief's input-output table for the six sectors of the economy of the United States for 1958. For example the value 0.018 in the FM row and the BM column means that \$0.018 of final metal output is needed as input to produce \$1 worth of basic metal output.

		to produce \$1 worth of output					
		FN	FM	BM	BN	E	S
Input needed...	FN	0.170	0.004	0	0.029	0	0.008
	FM	0.003	0.295	0.018	0.002	0.004	0.016
	BM	0.025	0.173	0.460	0.007	0.011	0.017
	BN	0.384	0.037	0.021	0.403	0.011	0.048
	E	0.007	0.001	0.039	0.025	0.358	0.025
	S	0.120	0.074	0.104	0.123	0.173	0.234

The total input, output and surplus can each be represented by a 6×1 matrix. For the 1958 data, these values would be given in millions of dollars. When the values of one of these matrices are given, the values for the other matrices can be determined.

FN	
FM	
BM	
BN	
E	
S	

To explain this, let the technology matrix = A , the surplus (or demand) matrix = D , the input (or supply) matrix = S , the output (or production schedule) matrix = P , and of course the identity matrix = I .

The output minus the input is the surplus $P - S = D$,
 and the input is equal to the technology times the output $S = A \times P$;
 therefore, $P - (A \times P) = D$.
 Factoring out the output matrix gives $P \times (I - A) = D$.
 Solve for the output by multiplying the inverse: $P = (I - A)^{-1} \times D$.

Example 3:

Use Excel to find the output of the U.S. 1958 economic input-output system. Enter the data for the technology matrix and a desired surplus matrix. Also enter a place to calculate the output (also formatted as currency) and a 6x6 identity matrix.

	A	B	C	D	E	F	G	H	I	J
1	input output	FN	FM	BM	BN	E	S		Demand	Output
2	FN	0.170	0.004	0	0.029	0	0.008		\$100,000	
3	FM	0.003	0.295	0.018	0.002	0.004	0.016		\$600,000	
4	BM	0.025	0.173	0.460	0.007	0.011	0.017		\$500,000	
5	BN	0.384	0.037	0.021	0.403	0.011	0.048		\$800,000	
6	E	0.007	0.001	0.039	0.025	0.358	0.025		\$200,000	
7	S	0.120	0.074	0.104	0.123	0.173	0.234		\$100,000	
8										
9	1	0	0	0	0	0				
10	0	1	0	0	0	0				
11	0	0	1	0	0	0				
12	0	0	0	1	0	0				
13	0	0	0	0	1	0				
14	0	0	0	0	0	1				

To calculate the output for each of the economic sectors using the matrix equation above, highlight a 6x1 matrix (J2:J7 in this example) for the answer. Click the formula bar above the spreadsheet which has the function symbol fx outside. Click the fx to get the Insert Function dialogue box. Select the category: Trig & Math. To get the order of operations correct, they must be entered in the opposite order from which they will be performed. Scroll down to select the function MMULT and click OK. The Function Arguments dialogue box appears to receive the arrays to be multiplied. Array 1 will have the inverse matrix function, and it must be entered manually. Type “MINVERSE(”, then drag the curser through the identity matrix A9:F14. Type the subtraction sign, drag the curser through the technology matrix, and finish the parenthesis. Array 1 should contain the argument MINVERSE(A9:F14-B2:G7). In Array 2, drag the curser through the demand values I2:I7, but do not click OK. To perform the matrix function, hit Ctrl, Shift, and Enter all at the same time.

The selected demand values and the calculated output values for each of the six economic sectors should be:

	A		I	J
1	input	output	Demand	Output
2	FN		\$100,000	\$189,711
3	FM		\$600,000	\$910,071
4	BM		\$500,000	\$1,282,450
5	BN		\$800,000	\$1,636,494
6	E		\$200,000	\$487,611
7	S		\$100,000	\$795,210
8				

Assignment:

4. Use Leontief's input-output table for the six sectors of the economy of the United States for 1958 to find the production schedule (total outputs) required to fulfill each of the final demands below.

Sector	Final demand A	Final demand B	Final demand C	Final demand D
FN	100,000	300,000	30,000	10,000
FM	100,000	200,000	20,000	60,000
BM	100,000	400,000	50,000	50,000
BN	100,000	100,000	10,000	80,000
E	100,000	100,000	10,000	20,000
S	100,000	900,000	90,000	10,000

Print each of the demand values and the calculated output values for each of the six economic sectors as in the example.

5. Write a paragraph telling what you have observed in the use of matrices to work and understand applied problems.

¹ Based on problems #27 and #28, pp. 486-7, Harshbarger & Yocco, College Algebra in Context, 2nd ed.

² Based on problem #33, p. 488, Harshbarger & Yocco, College Algebra in Context, 2nd ed.

³ Based on problem #36, p. 488, Harshbarger & Yocco, College Algebra in Context, 2nd ed.

⁴ Adapted from a case study on the Leontief model of the economy, from Finite Mathematics with Calculus, an Applied Approach, by Coughlin and Zitarelli.